

TECHNICAL REVIEW ON RANGING AND ODOMETRY FOR ANY ROBOTS

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ABSTRACT

This chapter examines how certain properties of the world can be exploited in order for a robot or other device to develop a model of its own motion or pose (position and orientation) relative to an external frame of reference. Although this is a critical problem for many autonomous robotic systems, the problem of establishing and maintaining an orientation or position estimate of a mobile agent has a long history in terrestrial navigation.

ODOMETRY

The word odometry is a contraction of the Greek words hodos meaning travel or journey, and metron meaning measure. Given its importance to a wide variety of applications from civil engineering to military conquest, the basic concepts that underly odometry haven been studied for over 2000 years. Perhaps the earliest reference to odometry appears in the Ten Books on Architecture by Vitruvius, in which he describes a useful invention of the greatest ingenuity, transmitted by our predecessors, which enables us, while sitting in a carriage on the road or sailing by sea, to know how many miles of a journey we have accomplished. In the context of autonomous vehicles, odometry usually refers to the use of data from the actuators (wheels, treads, etc.) to estimate the overall motion of the vehicle. The basic concept is to develop a mathematical model of how selected motions of the vehicle's wheels, joints, etc. induce motion of the vehicle itself, and then to integrate these specified motions over time in order to develop a model of the pose of the vehicle as a function of time. The use of odometry information to estimate the pose of the vehicle as a function of time is know as dead reckoning or deductive reckoning and finds wide application in navigation at sea.

The details of odometry estimation varies by vehicle design. In the context of mobile robots perhaps the simplest vehicle for odometry estimation is the differential drive vehicle. A differential drive vehicle has two driveable wheels which are independently controllable and which are mounted along a common axis. Assuming that the location of the wheels are fixed on the vehicle, then for the wheels to remain in constant contact with the ground, the two wheels must describe arcs on the plane such that the vehicle rotates around a point (known as the ICC – instantaneous center of curvature) that lies on the wheels' common axis. If the ground contact speeds of the left and right wheels are v_l and v_r respectively, and the wheels are separated by a distance $2d$, then

$$\omega(R+d) = v_l$$

$$\omega(R-d) = v_r$$

We can rearrange these two equations to solve for ω the rate of rotation about the ICC and R the distance from the center of the robot to the ICC

$$\omega = \frac{(v_l - v_r)}{2d}$$

$$R = d \frac{(v_r + v_l)}{2d(v_l - v_r)}$$

The instantaneous velocity of the point midway between the robot's wheels is given by $V = \omega R$. Now as v_l and v_r are functions of time we can generate a set of equations of motion for the differential drive robot. Using the point midway between the wheels as the origin of the robot, and writing θ as the orientation of the robot with respect to the x-axis of a global Cartesian coordinate system, This is the solution for the odometry of a differential drive vehicle on the plane. Given the control inputs (v_l and v_r) and some initial state estimate, we can estimate the state of an idealized robot using this motion model at any time t . Given such a model and complete knowledge of the control inputs, we should, in principle, be able to estimate a robot's pose at any time. In a perfect world this would be all that is necessary to estimate accurately the robot's pose at any time in the future.

Unfortunately errors in the modeling (incorrect estimations of wheel size, vehicle size), uncertainty about the control inputs, realities of the motor controller (errors between commanded wheel rotation and true rotation), errors in the physical modeling of the robot (wheel compaction, ground compaction, wheel slippage, nonzero tire width), etc., introduce an error between the dead reckoning estimate of the vehicle motion and its true motion. The problem of correcting for this error is the problem of pose maintenance for the vehicle, and requires the integration of the dead reckoning estimate with estimates obtained from other sensor systems. Other chapters in this handbook examine sensors that rely on external events, visual and otherwise, that can provide information as to the robot's pose or changes in its pose. Here we consider sensors that transduce physical properties of matter under the influence of external forces and properties of matter and the use of a global position system (GPS).

GYROSCOPIC SYSTEMS

The goal of gyroscopic systems is to measure changes in vehicle orientation by taking advantage of physical laws that produce predictable effects under rotation. A rotating frame is not an inertial frame, and thus many physical systems will appear to behave in an apparently non-Newtonian manner. By measuring these deviations from what would be expected in a Newtonian frame the underlying self-rotation can be extracted.

Mechanical Systems

Mechanical gyroscopes and gyrocompasses have a long history in navigation, Bohnenberger is generally credited with the first recorded construction of a gyroscope, and in 1851 Léon Foucault recognized the gyroscope as an inertial frame. The gyrocompass was patented in 1885 by Martinus Gerardus van den Bos. In 1903 Herman Anschütz - Kaempfe constructed a working gyrocompass and obtained a patent on the design. In 1908 Elmer Sperry patented a gyrocompass in the US and then attempted to sell this device to the German Navy. A patent battle followed, and Albert Einstein testified in the case. Gyroscopes and gyrocompasses rely on the principle of the conservation of angular momentum. Angular momentum is the

tendency of a rotating object to keep rotating at the same angular speed about the same axis of rotation in the absence of an external torque. The angular momentum \mathbf{L} of an object with moment of inertia \mathbf{I} rotating at angular speed ω is given by

$$\mathbf{L} = \mathbf{I} \times \omega$$

Consider a rapidly spinning wheel mounted on a shaft so that it is free to change its axis of rotation. Assuming no friction due to air resistance or the bearings, the rotor axis will remain constant regardless of the motion of the external cage. This constancy of orientation can be exploited to maintain a bearing independently of the motion of the vehicle, although it is not usually desirable to use the principle of conservation of angular momentum via a gyroscope directly. To see this, suppose that a gyroscope is set on the equator, with its spinning axis aligned along the equator. As the Earth spins, the gyroscope will maintain a constant axis of orientation and thus to an Earth-fixed observer will appear to rotate, returning to its original orientation every 24 h. Similarly, if the gyroscope were to be positioned on the equator such that its spinning axis was parallel to the axis of rotation of the earth, the gyroscope's axis of rotation would remain stationary and would appear to remain stationary to an Earth-fixed observer as the planet rotates.

Although this global motion limits the mechanical gyroscope's ability to sense absolute orientation directly, gyroscopes can be used to measure local changes in orientation, and thus are well suited to vehicular robotic applications. Rate gyros (RGs) measure a vehicle's rotation rate (its angular rate of rotation). This is the fundamental measurement that is the basis of all gyroscopic systems. Rate-integrating gyros (RIGs) use embedded processing to internally integrate the angular rotation rate to produce an estimate of the absolute angular displacement of the vehicle.

Optical Systems

Optical gyroscopes rely on the Sagnac effect rather than rotational inertia in order to measure (relative) heading. The mechanism is based on the behavior of an optical standing wave in a rotating frame. Historically this was first produced using lasers and an arrangement of mirrors, but it is now typically obtained using fibre optic technology. The Sagnac effect is named after its discoverer Georges Sagnac. The underlying concept can be traced back even earlier to the work of Harress, and perhaps finds its most famous application in terms of the measurement of the rotation of the Earth.

If two light pulses are sent in opposite directions around a stationary path of perimeter $D = 2\pi R$ they will travel the same distance at the same speed. They will arrive at the starting point simultaneously, taking time $t = D/c$ (where c is the speed of light in the medium). Now let us suppose that instead of being stationary, this circular light path rotates clockwise about its center at rotational speed ω . The light traveling clockwise around the path must go farther to reach the starting point, while light traveling counter clockwise around the path goes a shorter distance. The clockwise path has distance $D_c = 2\pi R + \omega R t_c$, where t_c is the time taken in the clockwise direction, while the counter clockwise path has distance $D_a = 2\pi R - \omega R t_a$, where t_a is the time taken in the counter clockwise direction. But $D_c = c t_c$ and $D_a = c t_a$, so t_c

$= 2\pi R/(c-\omega R)$ and $t_a = 2\pi R/(c+\omega R)$. In optical gyroscopes lasers are typically used as the light source.

Optical gyroscopes either employ straight line light paths with mirror surfaces or prisms at the edges to direct the light beam (a ring laser gyroscope – RLG), or a polarization maintaining glass-fiber loop (fiber optic gyro – FOG). The glass fiber may actually loop multiple times, thus extending the effective length of the light path. The time delay between the clockwise and counter clockwise directions is detected by examining the phase interference between the clockwise and counter clockwise light signals. Multiple optical gyroscopes with nonparallel axes can be ganged together in order to measure three-dimensional (3-D) rotations. Various techniques can be used to measure the time difference between the two paths, including examining the Doppler (frequency) shift of the laser light due to the motion of the gyro and an examination of the beat frequency of the interference pattern between the clockwise and counterclockwise paths. Ring interferometers typically consist of many windings of fiber-optic lines that conduct light of a fixed frequency in opposite directions around the loop and measure the phase difference. A ring laser consists of a laser cavity in the shape of a ring.

Light circulates in both directions around this cavity, producing two standing waves with the same number of nodes in both directions. Since the optical path lengths are different in the two directions, the resonant frequencies differ. The difference between these two frequencies is measured. An unfortunate side-effect of the ring-laser approach is that the two signals will lock in to each other for small rotations and it is typically necessary to physically rotate the device in a controlled manner in order to ensure that this lock-in effect can be avoided.

CONCLUSION

With the exception of gyrocompasses, gyroscopes measure relative rotational motion around a single axis. They accomplish this measurement by exploiting physical properties of rotating frames of reference. Earlier technologies based on mechanical gyroscopes have given way to optical- and MEMS-based devices but the underlying principle remains unchanged: that rotating frames of reference show specific physical properties that can be measured to estimate the relative rotation. A problem common to all gyroscopes is that of drift. Each of the relative motion measurements is corrupted by an error process, and these errors accumulate over time. This, coupled with specific measurement errors associated with the individual gyroscope technologies, means that unless the error is corrected through reference to some alternate (external) measurement, the drift will eventually exceed the required accuracy of the measurement. As individual gyros only measure rotation about a single axis, it is common to gang multiple gyros together with orthogonal axes of sensitivity in order to measure 3-D rotations. These collections of gyros are often integrated with other sensors (compasses, accelerometers, etc.) in order to construct inertial measurement units (or IMUs).

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